Numerical and analytical study on instabilities of the line soliton of the KP I equation and related recurrent phenomena

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Abstract:

We report some numerical results on time-evolutions of the KP I equation under periodic boundary conditions, which are compared with exact solutions. It has been shown analytically that conventional line solitons are linearly unstable to long-wave transverse disturbances[1]. An analytical solution of the line soliton with a small unstable mode has been also obtained, which shows that it becomes another smaller line soliton and a periodic soliton (i.e. the array of the localized structures) [2,3]. On the other hand, time-evolutions of the line soliton with *general* disturbances are not available to us. A series of numerical simulations are conducted in order to clarify how the line soliton develops with the following more general disturbances: (a) one finite-amplitude unstable mode, (b) a small sine wave with one periodicity similar to the case treated in [4] and (c) two small unstable modes. The Fourier-Galerkin method is applied for spatial discretization and the fourth-order Runge-Kutta method based on the integrating factor method is devised. In case (a) when the disturbance is small enough, the result agrees with the analytical solution. As the magnitude of the disturbance is larger, ripples grow and the remaining line soliton is deformed. In case (b) the time-evolution is very similar to case (a), but both solitons are a little different from those in case (a) with a very small unstable mode. In case (c) two periodic solitons and another line soliton appear after the line soliton is destroyed by the disturbances. The shape of the resulting solitons is independent of the ratio of the magnitude of two given modes though the processes of the time-evolution are different. The solitons interact with each other owing to the periodic boundary conditions. If the parameters of two solitons are satisfied with the resonant condition[5,6], they fuse into a soliton. Furthermore if the parameters of the fused soliton and the other soliton are satisfied with the resonant condition, they fuse into the initial *linearly unstable* line soliton, which is regarded as a recurrent phenomenon. A numerical simulation shows that the recurrent soliton appears after some interactions. The solitons appeared in passing numerically are in good agreement with the exact soliton solutions that are expected based on the resonant condition. The linearly unstable soliton may correspond to the fixed point of homoclinic orbits.

References:

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